





10. Intro to Geometry/  
Angles, Triangles/ Basic  
Word Problems

Name : \_\_\_\_\_

## Points, Lines and Planes

	Description	Figure	Symbol
<b>Point</b>	A geometric element that has zero dimensions.	• P	P or Point P
<b>Line</b>	A line is a collection of points along a straight path with no end points.		$\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$
<b>Line segment</b>	A line segment is a part of a line that contains every point on the line between its end points.		$\overline{XY}$ or $\overline{YX}$
<b>Ray</b>	A ray is a line with a single end point that goes on and on in one direction.		$\overrightarrow{PQ}$
<b>Plane</b>	A plane is a flat surface that extends to infinity.		Plane EFG or Plane $\tau$

W

**Basic Elements of Geometry**

PLP 1

**Instructions:** Match each basic element of geometry with the correct picture.

1  $\overleftrightarrow{CD}$

2 Line AB

3 Ray AB

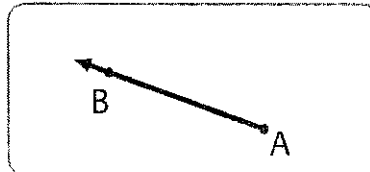
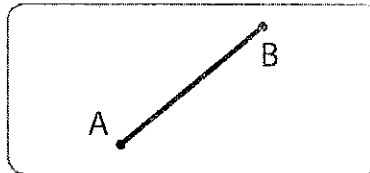
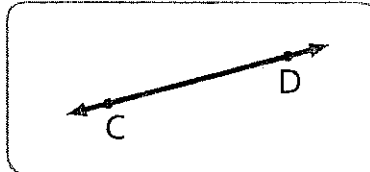
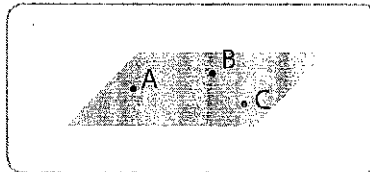
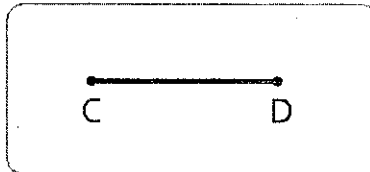
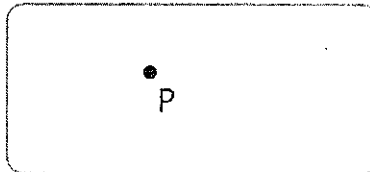
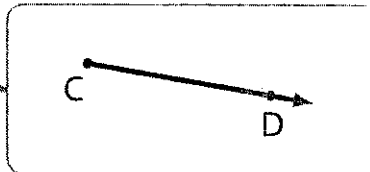
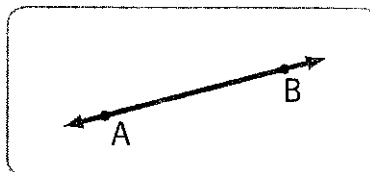
4  $\overline{CD}$

5 Point P

6 Line Segment AB

7 Plane ABC

8  $\overleftrightarrow{BA}$



**Basic Elements of Geometry**

PLP 1

**Instructions:** Match each basic element of geometry with the correct picture.

1  $\overleftrightarrow{CD}$

2 Line AB

3 Ray AB

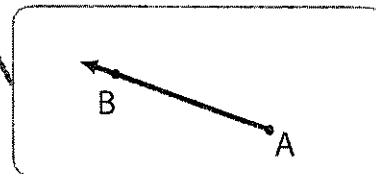
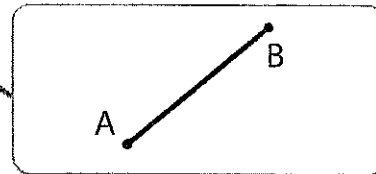
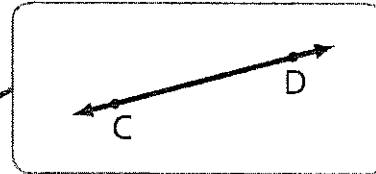
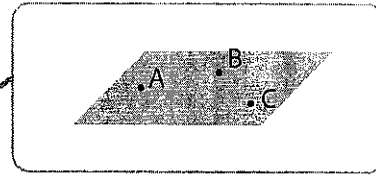
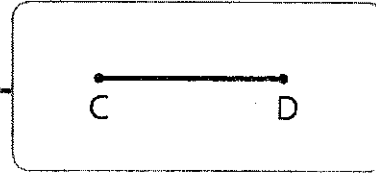
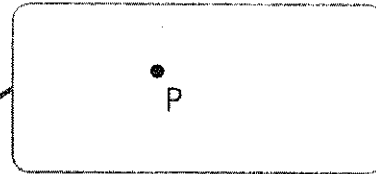
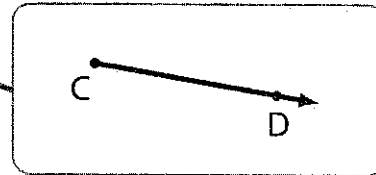
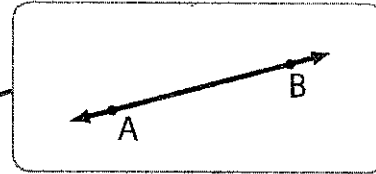
4  $\overline{CD}$

5 Point P

6 Line Segment AB

7 Plane ABC

8  $\overleftrightarrow{CD}$



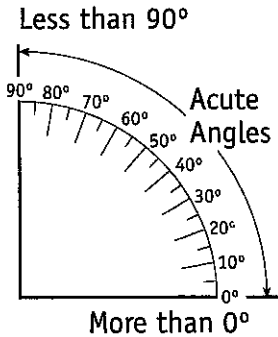
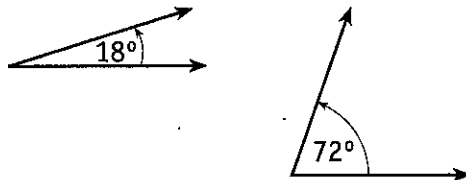
# Types of Angles

An angle is often named according to its size. Below are listed five types of angles with which you should become familiar.

## ACUTE ANGLES

An **acute angle** measures more than  $0^\circ$  but less than  $90^\circ$ .

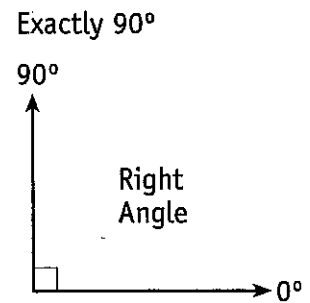
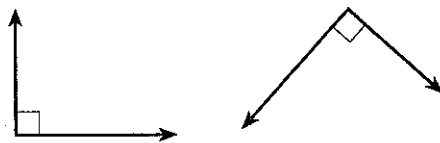
**EXAMPLES**



## RIGHT ANGLES

A **right angle** measures exactly  $90^\circ$ . A right angle is often indicated by placing a small square at its vertex.

**EXAMPLES**

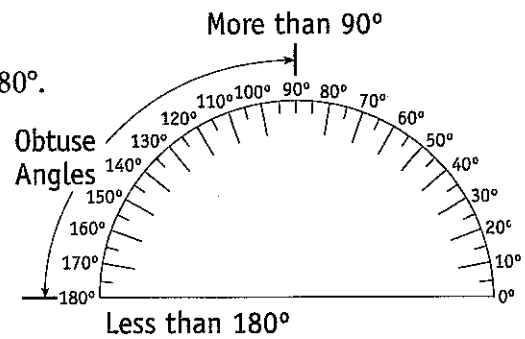
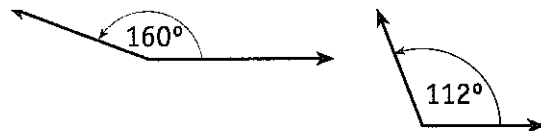


**Note:** Lines that meet at a right angle are called perpendicular lines. The symbol for perpendicular lines is  $\perp$ .

## OBTUSE ANGLES

An **obtuse angle** measures more than  $90^\circ$  but less than  $180^\circ$ .

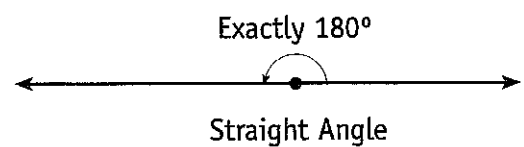
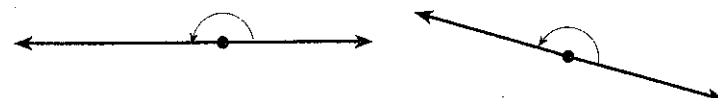
**EXAMPLES**



## STRAIGHT ANGLES

A **straight angle** measures exactly  $180^\circ$ . A straight angle is the same as a straight line.

**EXAMPLES**





# Pairs of Angles

As you've seen, a special name is given to an angle according to its size. Also, names are given to special pairs of angles. The names and relationships you should become familiar with are **complementary angles**, **supplementary angles**, and **vertical angles**.

## COMPLEMENTARY ANGLES

**Complementary angles** are angles whose sum is  $90^\circ$ .

In the drawing at the right, line TR divides right angle QRS into the two complementary angles,  $\angle QRT$  and  $\angle TRS$ .

$\angle QRT$  is called the **complement** of  $\angle TRS$ .

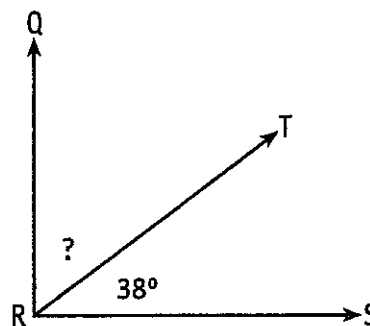
Often, the value of one complementary angle is given, and you are asked to find the value of the other.

**EXAMPLE** Find  $\angle QRT$  if  $\angle TRS = 38^\circ$ .

To find the value of a complementary angle, subtract the known angle from  $90^\circ$ .

$$\angle QRT = 90^\circ - 38^\circ = 52^\circ$$

**ANSWER:**  $\angle QRT = 52^\circ$



$\angle QRT$  and  $\angle TRS$   
are *complementary* angles.  
 $\angle QRT + \angle TRS = 90^\circ$

## SUPPLEMENTARY ANGLES

**Supplementary angles** are angles whose sum is  $180^\circ$ .

In the drawing at right, line RC divides the straight angle BCD into the two supplementary angles  $\angle BCR$  and  $\angle RCD$ .

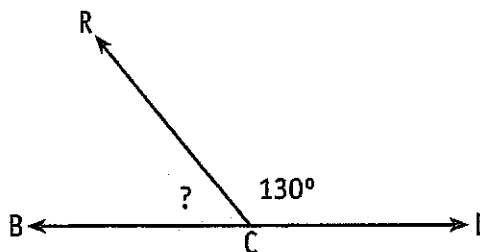
$\angle BCR$  is called the **supplement** of  $\angle RCD$ .

**EXAMPLE** Find  $\angle BCR$  if  $\angle RCD = 130^\circ$ .

To find the value of a supplementary angle, subtract the value of the known angle from  $180^\circ$ .

$$\angle BCR = 180^\circ - 130^\circ = 50^\circ$$

**ANSWER:**  $\angle BCR = 50^\circ$



$\angle BCR$  and  $\angle RCD$   
are *supplementary* angles.  
 $\angle BCR + \angle RCD = 180^\circ$

# Pairs of Angles: Applying Your Skills

Practical problems involving pairs of angles often require you to find an unknown angle. Examples that frequently appear include

- finding an angle between a ladder and the ground
- finding an angle between two boards or beams
- finding an angle between intersecting streets
- finding a missing corner-of-the-room angle

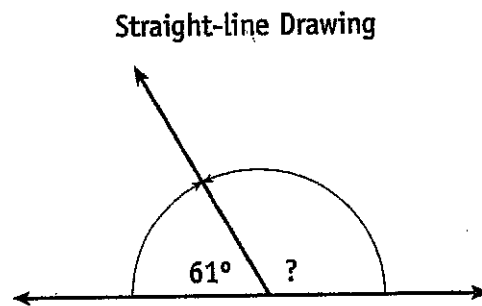
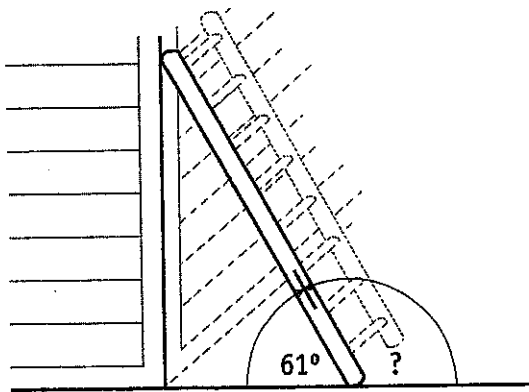
In each case, you are really being asked to find the complement or supplement of a given angle or to recognize an equal vertical angle.

In books and on tests, practical problems often appear as word problems. You may or may not be given an accompanying drawing. In either case, the first step in solving a word problem is to represent the information in the problem by a drawing that contains only straight lines. Then, solve the straight-line drawing by using the skills you learned on the previous few pages.

**EXAMPLE** A ladder leans against the side of a house. The ladder makes an acute angle of  $61^\circ$  with the ground. What is the measure of the obtuse angle the ladder makes with the ground?

In this word problem, no drawing is given.

For comparison, look at both an accurate picture and a straight-line drawing.



Notice that the use of a straight-line drawing simplifies the problem by focusing attention on the unknown angle. The question really asks,

"What is the supplement of  $61^\circ$ ?"

Find the supplement of  $61^\circ$  by subtracting  $61^\circ$  from  $180^\circ$ .

$$\text{Supplement of } 61^\circ = 180^\circ - 61^\circ = 119^\circ$$

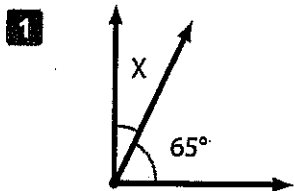
**ANSWER:** The obtuse angle is  $119^\circ$ .



**Finding an Unknown Angle**

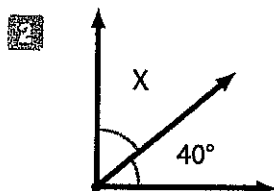
AAD 3

**Instructions:** For each set of complementary or supplementary angles, find the unknown angle (X).

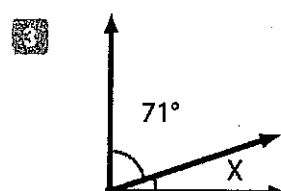


$m\angle X = 25^\circ$

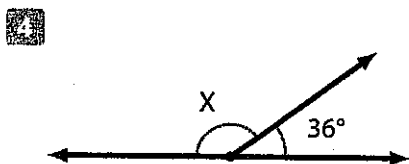
$$\begin{array}{r} 90 \\ - 65 \\ \hline 25 \end{array}$$



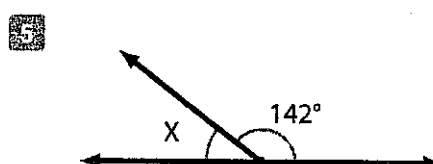
$m\angle X = \underline{\hspace{2cm}}$



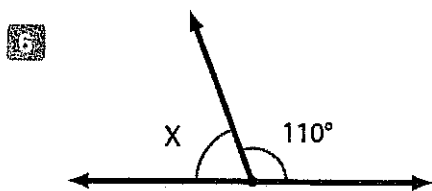
$m\angle X = \underline{\hspace{2cm}}$



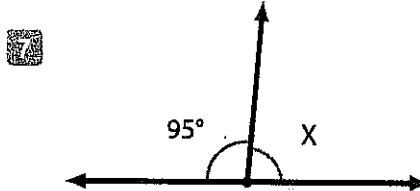
$m\angle X = \underline{\hspace{2cm}}$



$m\angle X = \underline{\hspace{2cm}}$



$m\angle X = \underline{\hspace{2cm}}$



$m\angle X = \underline{\hspace{2cm}}$

**Finding an Unknown Angle**

AAD 3

**Instructions:** For each set of complementary or supplementary angles, find the unknown angle (X).

**1**

$m\angle X = \underline{25^\circ}$

$$\begin{array}{r} 8 \\ 90 \\ - 65 \\ \hline 25 \end{array}$$

**2**

$m\angle X = \underline{50^\circ}$

$$\begin{array}{r} 90 \\ - 40 \\ \hline 50 \end{array}$$

**3**

$m\angle X = \underline{19^\circ}$

$$\begin{array}{r} 8 \\ 90 \\ - 71 \\ \hline 19 \end{array}$$

**4**

$m\angle X = \underline{144^\circ}$

$$\begin{array}{r} 7 \\ 180 \\ - 36 \\ \hline 144 \end{array}$$

**5**

$m\angle X = \underline{38^\circ}$

$$\begin{array}{r} 7 \\ 180 \\ - 142 \\ \hline 38 \end{array}$$

**6**

$m\angle X = \underline{70^\circ}$

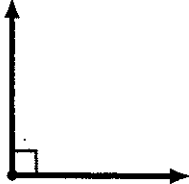

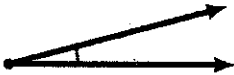

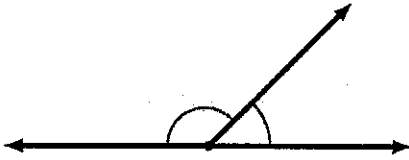

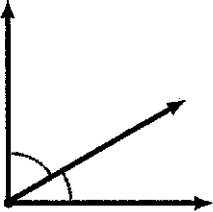
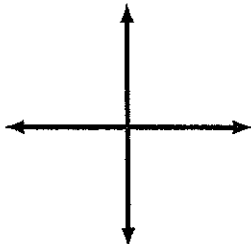
$$\begin{array}{r} 180 \\ - 110 \\ \hline 70 \end{array}$$

**7**

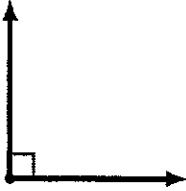
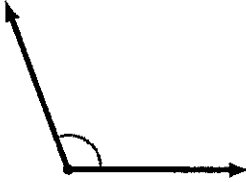


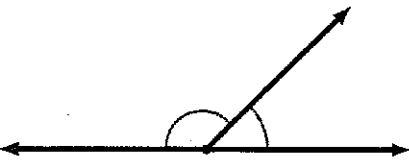
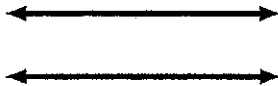
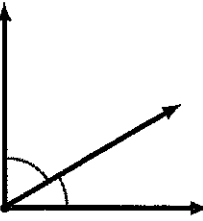
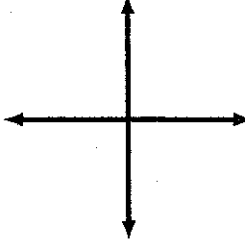
$m\angle X = \underline{85^\circ}$

$$\begin{array}{r} 17 \\ 180 \\ - 95 \\ \hline 85 \end{array}$$

**Angle Basics**

<p><b>1</b> What kind of angle is this? <u>right</u></p> 	<p><b>2</b> What kind of angle is this? <u>obtuse</u></p> 
<p><b>3</b> What kind of angle is this? <u>acute</u></p> 	<p><b>4</b> What kind of angle is this? <u>straight</u></p> 
<p><b>5</b> This diagram shows:</p>  <p> <input type="checkbox"/> Parallel Lines  <input type="checkbox"/> Perpendicular Lines  <input checked="" type="checkbox"/> Supplementary Angles  <input type="checkbox"/> Complementary Angles         </p>	<p><b>6</b> This diagram shows:</p>  <p> <input checked="" type="checkbox"/> Parallel Lines  <input type="checkbox"/> Perpendicular Lines  <input type="checkbox"/> Supplementary Angles  <input type="checkbox"/> Complementary Angles         </p>
<p><b>7</b> This diagram shows:</p>  <p> <input type="checkbox"/> Parallel Lines  <input type="checkbox"/> Perpendicular Lines  <input type="checkbox"/> Supplementary Angles  <input checked="" type="checkbox"/> Complementary Angles         </p>	<p><b>8</b> This diagram shows:</p>  <p> <input type="checkbox"/> Parallel Lines  <input checked="" type="checkbox"/> Perpendicular Lines  <input type="checkbox"/> Supplementary Angles  <input type="checkbox"/> Complementary Angles         </p>

## Angle Basics

<p><b>1</b> What kind of angle is this? _____</p> 	<p><b>2</b> What kind of angle is this? _____</p> 
<p><b>3</b> What kind of angle is this? _____</p> 	<p><b>4</b> What kind of angle is this? _____</p> 
<p><b>5</b> This diagram shows:</p>  <ul style="list-style-type: none"> <li><input type="checkbox"/> Parallel Lines</li> <li><input type="checkbox"/> Perpendicular Lines</li> <li><input type="checkbox"/> Supplementary Angles</li> <li><input type="checkbox"/> Complementary Angles</li> </ul>	<p><b>6</b> This diagram shows:</p>  <ul style="list-style-type: none"> <li><input type="checkbox"/> Parallel Lines</li> <li><input type="checkbox"/> Perpendicular Lines</li> <li><input type="checkbox"/> Supplementary Angles</li> <li><input type="checkbox"/> Complementary Angles</li> </ul>
<p><b>7</b> This diagram shows:</p>  <ul style="list-style-type: none"> <li><input type="checkbox"/> Parallel Lines</li> <li><input type="checkbox"/> Perpendicular Lines</li> <li><input type="checkbox"/> Supplementary Angles</li> <li><input type="checkbox"/> Complementary Angles</li> </ul>	<p><b>8</b> This diagram shows:</p>  <ul style="list-style-type: none"> <li><input type="checkbox"/> Parallel Lines</li> <li><input type="checkbox"/> Perpendicular Lines</li> <li><input type="checkbox"/> Supplementary Angles</li> <li><input type="checkbox"/> Complementary Angles</li> </ul>

## Measuring an Angle with a Protractor

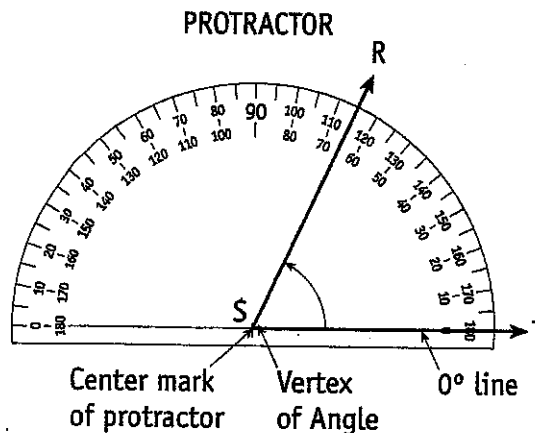
A **protractor** is a tool used for measuring and drawing angles. The most commonly used protractor is shaped like a half circle and can measure angles up to  $180^\circ$ . Carpenters, machinists, and others who read blueprints or plans use protractors in their work.

**EXAMPLE** Use a protractor to measure angle RST at the right.

**STEP 1** Place the center mark of the protractor at the vertex of the angle, and place the  $0^\circ$  line along one side (ST) of angle RST.

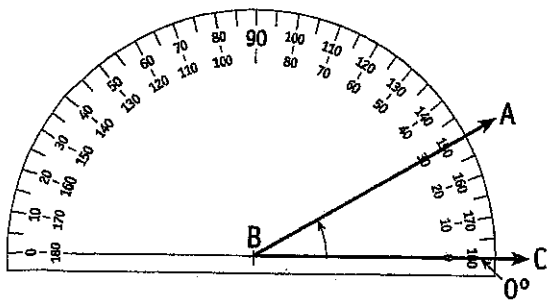
**STEP 2** Read the point where the second side (SR) crosses the protractor scale.

**ANSWER:**  $\angle RST = 65^\circ$

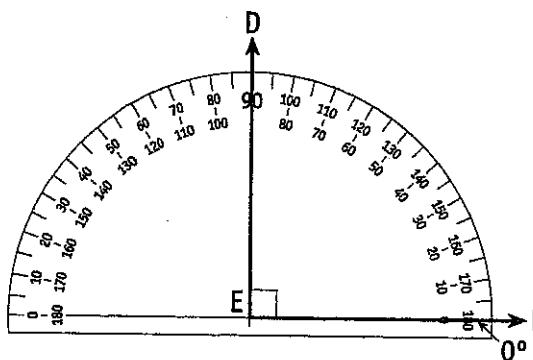


Measure and name the type of each angle drawn below.

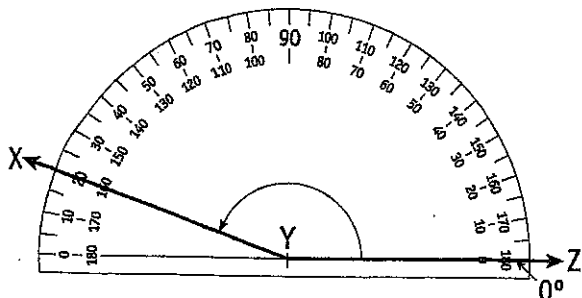
1.  $\angle ABC =$  \_\_\_\_\_  
Type of Angle: \_\_\_\_\_



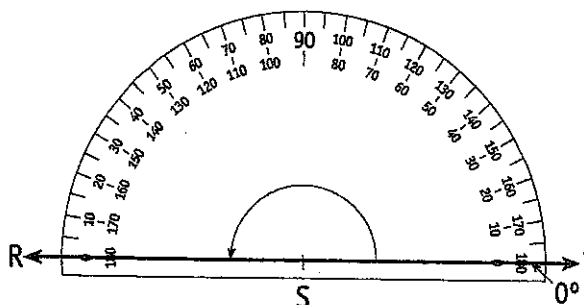
2.  $\angle DEF =$  \_\_\_\_\_  
Type of Angle: \_\_\_\_\_



3.  $\angle XYZ =$  \_\_\_\_\_  
Type of Angle: \_\_\_\_\_



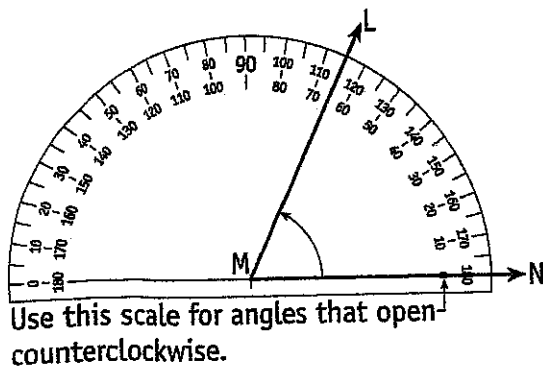
4.  $\angle RST =$  \_\_\_\_\_  
Type of Angle: \_\_\_\_\_



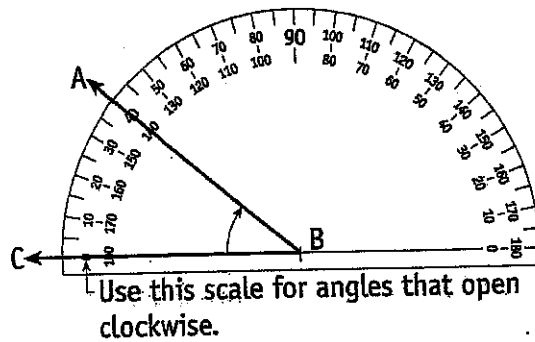
You can identify an angle by an arc that points either clockwise ↻ or counterclockwise ↺. So far in the study of angles, you have identified each angle as opening counterclockwise. However, many angles are easier to measure if you identify them as opening in a clockwise direction.

To make measuring easier, a protractor usually has two scales. The inside scale is used to measure angles that open counterclockwise, and the outside scale is used to measure angles that open clockwise. Be careful to read the correct scale for the angle you are measuring.

**EXAMPLE 1** Angle LMN is an acute angle that opens counterclockwise.  
 $\angle LMN$  measures  $65^\circ$



**EXAMPLE 2** Angle ABC is an acute angle that opens clockwise.  
 $\angle ABC$  measures  $40^\circ$

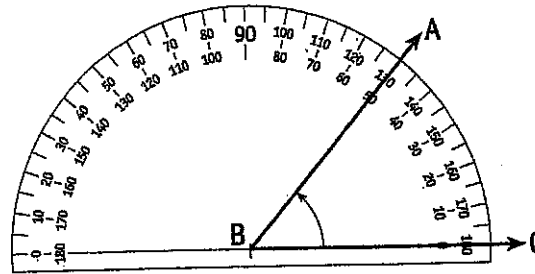


Measure each angle and circle the letter of the correct response.

5.  $\angle ABC$  measures \_\_\_\_\_.

The name given to  $\angle ABC$  is

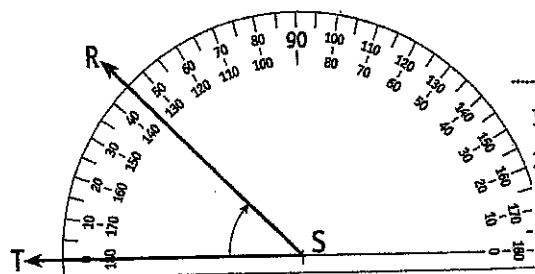
- a. acute
- b. right
- c. obtuse
- d. straight
- e. reflex



6.  $\angle RST$  measures \_\_\_\_\_.

The name given to  $\angle RST$  is

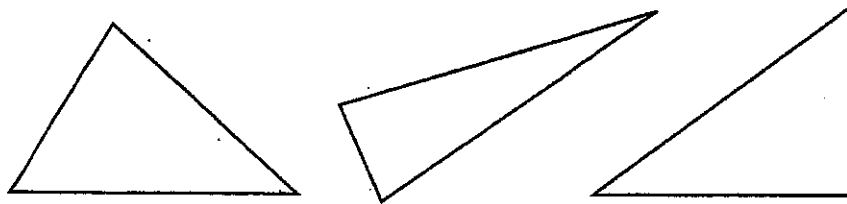
- a. acute
- b. right
- c. obtuse
- d. straight
- e. reflex



- 1.  $\angle ABC = 30^\circ$ , acute
- 2.  $\angle DEF = 90^\circ$ , right
- 3.  $\angle XYZ = 160^\circ$ , obtuse
- 4.  $\angle RST = 180^\circ$ , straight
- 5.  $\angle ABC = 50^\circ$ , a. acute
- 6.  $\angle RST = 45^\circ$ , a. acute

# Introducing Triangles

A **triangle** is a plane (flat) figure that has three sides and three angles. Triangles appear in a variety of shapes and sizes.



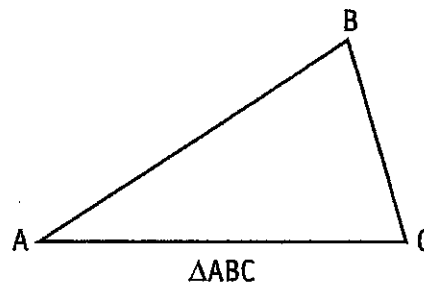
Triangles

## LABELING A TRIANGLE

The most common way to label a triangle is with three letters, usually written in alphabetical order. One letter is placed at the vertex of each angle.

The symbol for triangle is  $\Delta$ —a small triangle with three equal sides.

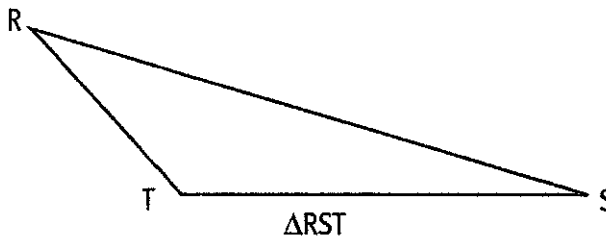
**EXAMPLE** The triangle at the right is represented as  $\Delta ABC$ .



## REPRESENTING A SIDE OF A TRIANGLE

The side of a triangle is represented by the two letters at the ends of that side.

**EXAMPLE** Side  $ST$  is the side *opposite* (across from)  $\angle R$ .  
In  $\Delta RST$ , side  $RS$  is the longest side.

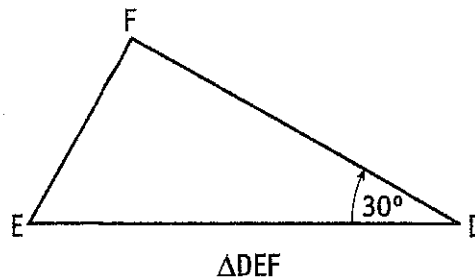


**Note:** The longest side of a triangle is always opposite the largest angle.

## REPRESENTING AN ANGLE IN A TRIANGLE

An angle in a triangle is most often represented by the angle's vertex letter only.

**EXAMPLE** In  $\Delta DEF$ , the  $30^\circ$  angle is labeled  $D$  and usually identified as  $\angle D$ .



**Note:** Angle  $D$  can also be called  $\angle EDF$  or  $\angle FDE$ . When three letters are used, the vertex letter is written second.

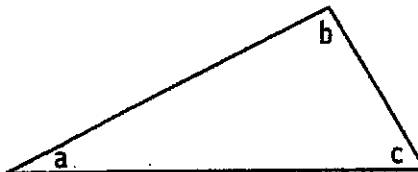
# The Sum of Angles in a Triangle

One of the most important rules to learn in the study of geometry is the following:

**The sum of the three angles in a triangle is equal to  $180^\circ$ .**

You can write this rule in symbols.

$$\angle a + \angle b + \angle c = 180^\circ$$

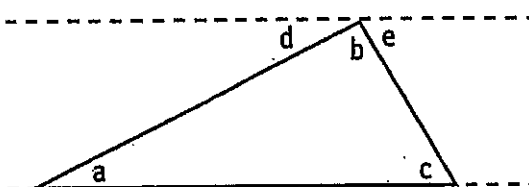


To see why this rule is true, enclose a triangle in parallel lines and use information that you learned in the chapter on angles.

You know that  $\angle d + \angle b + \angle e = 180^\circ$  since together they make up a straight angle.

Also, you know that  $\angle a = \angle d$  and  $\angle c = \angle e$  since these are pairs of opposite interior angles.

Thus, the sum  $\angle a + \angle b + \angle c$  is equal to  $\angle d + \angle b + \angle e$ , which is  $180^\circ$ .



$$\angle a + \angle b + \angle c = 180^\circ$$

The rule stated above is commonly used to find the measure of the third angle in a triangle when the other two are known. To find the measure of the third angle, subtract the sum of the two known angles from  $180^\circ$ .

**EXAMPLE 1** What is the measure of  $\angle B$  in triangle ABC?

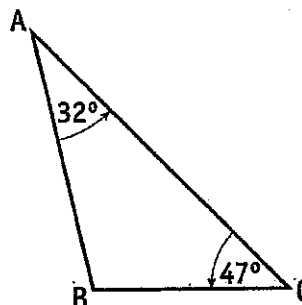
**STEP 1** Add  $\angle A$  and  $\angle C$ .

$$\angle A + \angle C = 32^\circ + 47^\circ = 79^\circ$$

**STEP 2** To find  $\angle B$ , subtract  $79^\circ$  from  $180^\circ$ .

$$\angle B = 180^\circ - 79^\circ = 101^\circ$$

**ANSWER:**  $\angle B = 101^\circ$



**EXAMPLE 2** What is the measure of  $\angle D$  in the right triangle DEF?

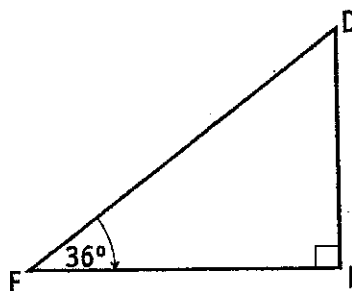
**STEP 1** Add  $\angle E$  and  $\angle F$ . Remember:  $\angle E = 90^\circ$ .

$$\angle E + \angle F = 90^\circ + 36^\circ = 126^\circ$$

**STEP 2** To find  $\angle D$ , subtract  $126^\circ$  from  $180^\circ$ .

$$\angle D = 180^\circ - 126^\circ = 54^\circ$$

**ANSWER:**  $\angle D = 54^\circ$

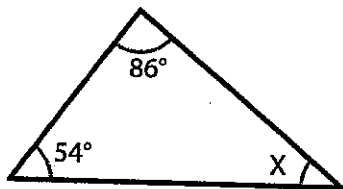




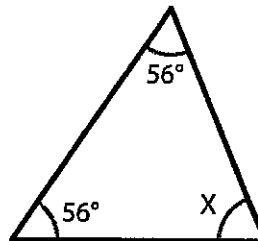
**Finding an Unknown Angle - Set 2**

TRI 5

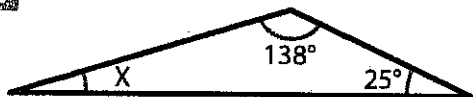
**Instructions:** For each triangle, find the unknown angle (X). Remember that for each triangle, the three interior angles must add up to 180 degrees.



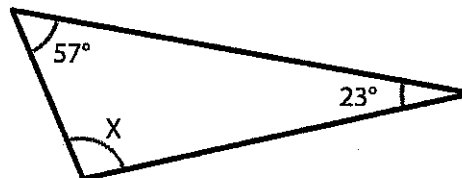
$m\angle X = \underline{\hspace{2cm}}$



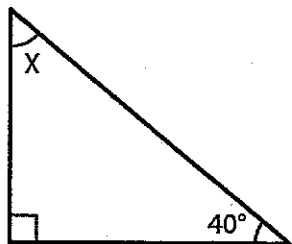
$m\angle X = \underline{\hspace{2cm}}$



$m\angle X = \underline{\hspace{2cm}}$



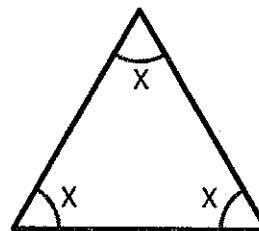
$m\angle X = \underline{\hspace{2cm}}$



$m\angle X = \underline{\hspace{2cm}}$



An equilateral triangle always has three equal angles. What is their measure?



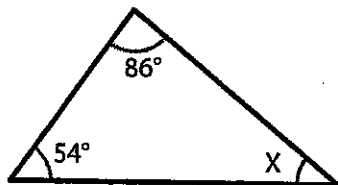
$m\angle X = \underline{\hspace{2cm}}$

Finding an Unknown Angle - Set 2

TRI 5

**Instructions:** For each triangle, find the unknown angle (X). Remember that for each triangle, the three interior angles must add up to 180 degrees.

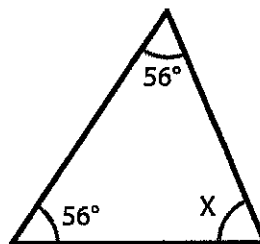
1



$m\angle X = \underline{40^\circ}$

$$\begin{array}{r} 86 \\ + 54 \\ \hline 140 \end{array} \quad \begin{array}{r} 180 \\ - 140 \\ \hline 40 \end{array}$$

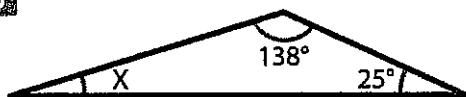
2



$m\angle X = \underline{68^\circ}$

$$\begin{array}{r} 56 \\ + 56 \\ \hline 112 \end{array} \quad \begin{array}{r} 180 \\ - 112 \\ \hline 68 \end{array}$$

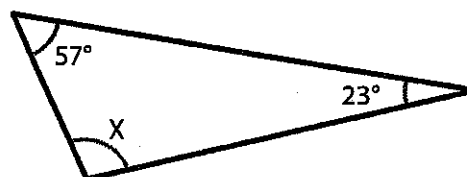
3



$m\angle X = \underline{17^\circ}$

$$\begin{array}{r} 138 \\ + 25 \\ \hline 163 \end{array} \quad \begin{array}{r} 180 \\ - 163 \\ \hline 17 \end{array}$$

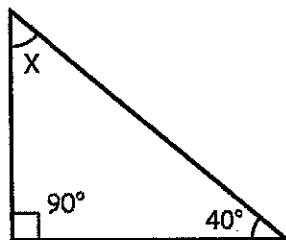
4



$m\angle X = \underline{100^\circ}$

$$\begin{array}{r} 57 \\ + 23 \\ \hline 80 \end{array} \quad \begin{array}{r} 180 \\ - 80 \\ \hline 100 \end{array}$$

5

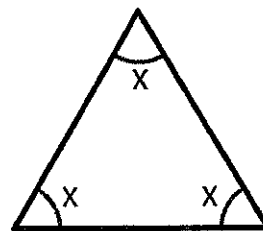


$m\angle X = \underline{50^\circ}$

$$\begin{array}{r} 90 \\ + 40 \\ \hline 130 \end{array} \quad \begin{array}{r} 180 \\ - 130 \\ \hline 50 \end{array}$$

6

An equilateral triangle always has three equal angles. What is their measure?



$m\angle X = \underline{60^\circ}$

To get the answer, divide the total (180°) by 3

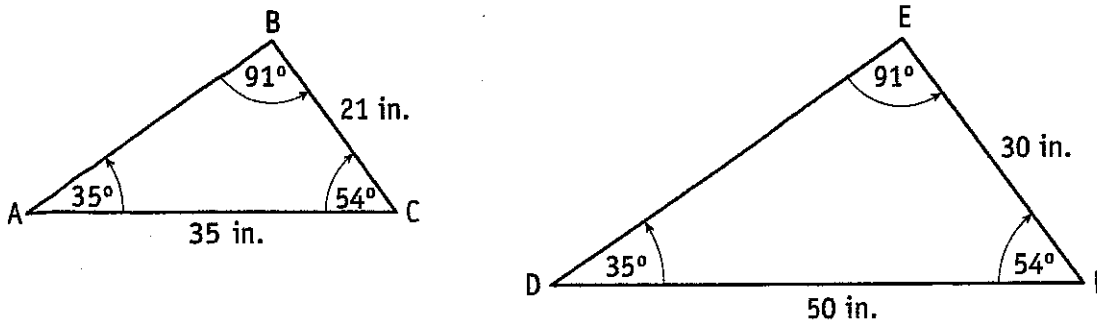
$$\begin{array}{r} 60^\circ \\ 3 \overline{)180} \\ \underline{-18} \\ 00 \end{array}$$

Similar triangles are interesting because the lengths of corresponding sides are in **proportion** and can be written as **equivalent fractions**.

In your earlier work with mathematics, you worked with equivalent fractions; for instance  $\frac{2}{4} = \frac{1}{2}$ . A proportion is an example of equivalent fractions. In fact, the study of similar triangles is a use of equivalent fractions in geometry.

Example 3 below illustrates how to write corresponding sides as a proportion.

**EXAMPLE 3** Because  $\triangle ABC$  and  $\triangle DEF$  have corresponding angles, they are similar triangles.



We can write a proportion as follows.

**STEP 1** Identify the pairs of corresponding sides.

- a. AC and DF are corresponding sides.
- b. BC and EF are corresponding sides.

**STEP 2** Write equivalent fractions (to be called a proportion from now on).

$$\frac{AC}{DF} = \frac{BC}{EF}$$

$$\frac{35}{50} = \frac{21}{30}$$

(To see that both sides of the proportion are equal, see that both can be reduced to  $\frac{7}{10}$ .)

In Example 3, it would have also been correct to write the proportion

$$\frac{DF}{AC} = \frac{EF}{BC}$$

$$\frac{50}{35} = \frac{30}{21}$$

In both cases, the corresponding sides are on the same side of the equals sign.



For each pair of similar triangles below, fill in the missing term for each proportion. The first problem is done as an example.

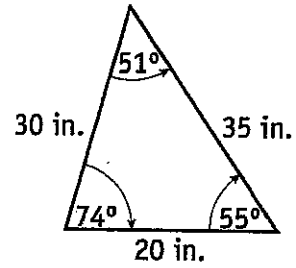
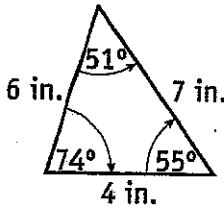
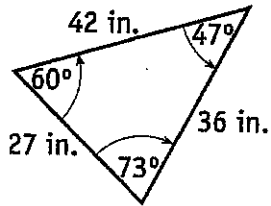
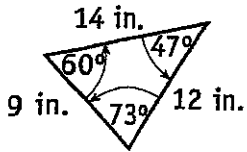
1. Problem

$$\frac{12}{36} = \frac{\quad}{27}$$

Answer

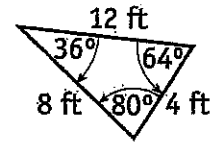
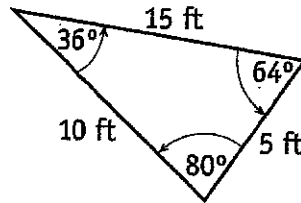
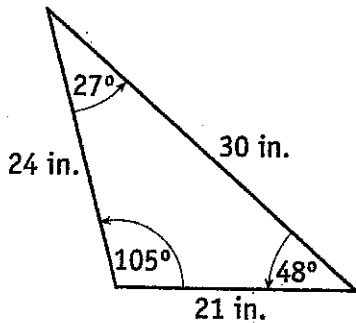
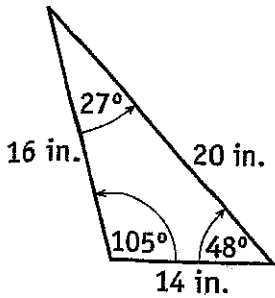
$$\frac{12}{36} = \frac{9}{27}$$

2.  $\frac{7}{35} = \frac{\quad}{20}$



3.  $\frac{\quad}{30} = \frac{14}{21}$

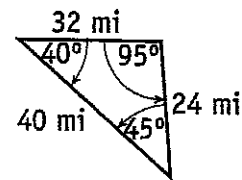
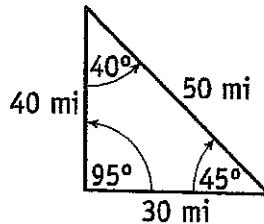
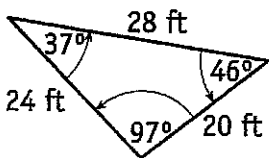
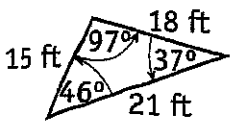
4.  $\frac{15}{12} = \frac{10}{\quad}$



In problems 5 and 6, don't be fooled because the triangles are turned. Carefully find corresponding sides before completing each proportion.

5.  $\frac{18}{\quad} = \frac{15}{20}$

6.  $\frac{40}{32} = \frac{\quad}{24}$



- 1. 9 in.
- 2. 4 in.
- 3. 20 in.
- 4. 8 ft
- 5. 24 ft
- 6. 30 mi

Example 4 shows how similar triangles can be used to find an unknown distance.

**EXAMPLE 4** What is length  $m$  of side LN in  $\triangle LMN$  below?

**STEP 1** Write a proportion using the corresponding sides of triangles RST and LMN.

$$\frac{12}{9} = \frac{8}{m}$$

**STEP 2** Solve the proportion by **cross multiplying**. To cross-multiply, follow steps a. and b.

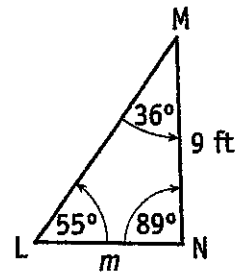
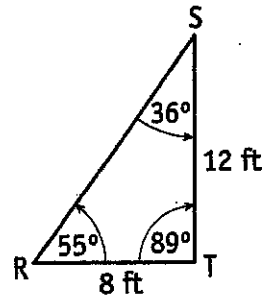
a. Multiply the numerator of each fraction by the denominator of the other fraction. Set the answers equal to each other.

$$\frac{12}{9} \times \frac{8}{m} = \frac{8}{m} \times \frac{12}{9}$$

$$12m = 72$$

b. Divide the number standing alone by the number next to the  $m$ . Since 12 times  $m$  equals 72, find  $m$  by dividing 72 by 12.

$$m = \frac{72}{12} = 6$$

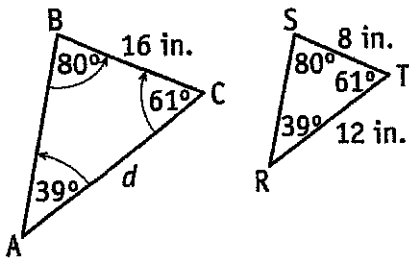


**ANSWER:** Side LN has a length of 6 feet.

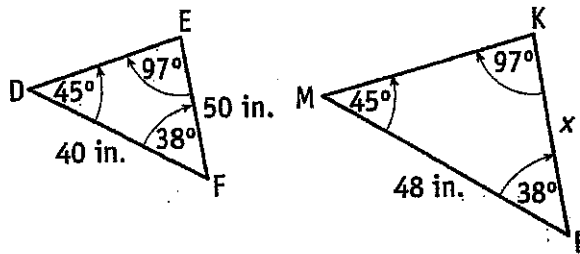


Find each length as indicated below.

1.  $\triangle ABC \sim \triangle RST$ . What is length  $d$ ?



2.  $\triangle DEF \sim \triangle KLM$ . What is length  $x$ ?

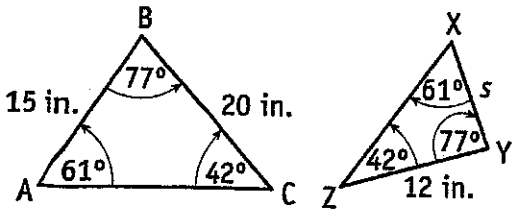


1.  $d = 24$  in.

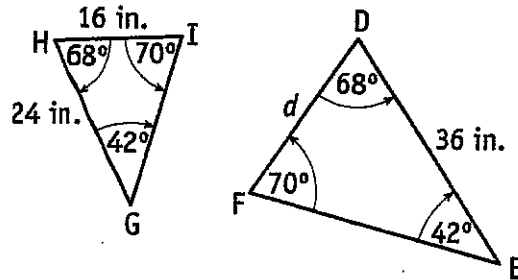
2.  $x = 60$  in.

In problems 3 through 8, find corresponding sides before completing each proportion.

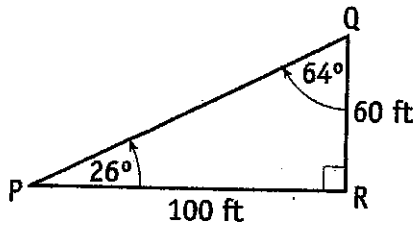
3.  $\triangle ABC \sim \triangle XYZ$ . What is length  $s$ ?



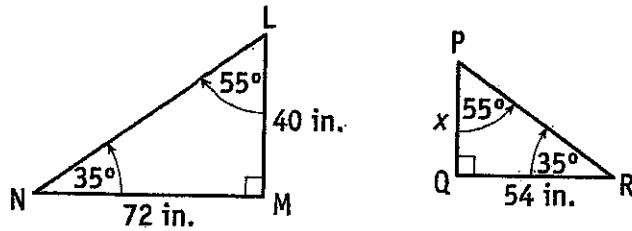
4.  $\triangle GHI \sim \triangle DEF$ . What is length  $d$ ?



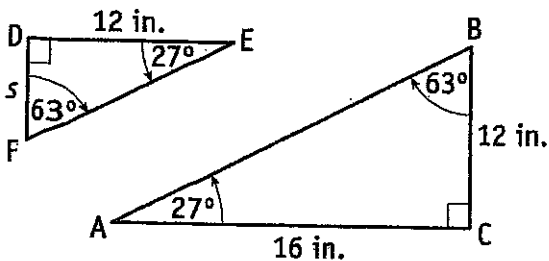
5.  $\triangle PQR \sim \triangle EFG$ . What is length  $b$ ?



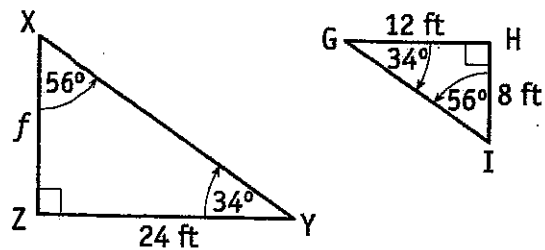
6.  $\triangle LMN \sim \triangle PQR$ . What is length  $x$ ?



7.  $\triangle ABC \sim \triangle DEF$ . What is length  $s$ ?



8.  $\triangle XYZ \sim \triangle GHI$ . What is length  $f$ ?



4.  $d = 24$  in.      7.  $s = 9$  in.

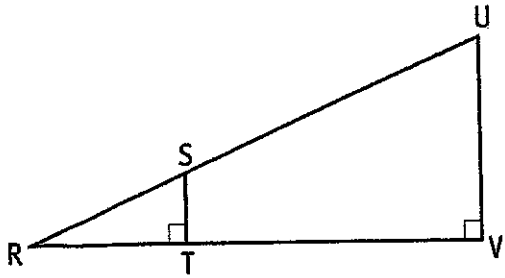
5.  $b = 36$  ft      8.  $f = 16$  ft

3.  $s = 9$  in.

6.  $x = 30$  in.

There are two geometric figures involving similar triangles that occur often in practical applications and on tests. Study the figures below in Examples 5 and 6 before working problems 1 through 4.

**EXAMPLE 5**  $\triangle RST \sim \triangle RUV$



Do you see why  $\triangle RST$  is similar to  $\triangle RUV$ ?

The reason is that each triangle has the same three angles.

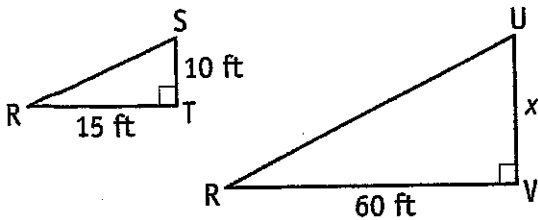
- a.  $\angle R$  is an angle in each triangle.
- b.  $\angle RTS = \angle RVU$ ; each is a right angle.
- c.  $\angle RST = \angle RUV$ . Since the two other angles are equal in both triangles, the third angle must also be the same.

**Sample Question:**

If  $ST = 10$  ft,  $RT = 15$  ft, and  $RV = 60$  ft, what is the length of  $UV$ ?

**Solution:**

**STEP 1** Many students find it helpful to draw the similar triangles separately.



**STEP 2** Write a proportion. Let  $x$  stand for the unknown value of  $UV$ .

$$\frac{UV}{ST} = \frac{RV}{RT} \text{ or } \frac{x}{10} = \frac{60}{15}$$

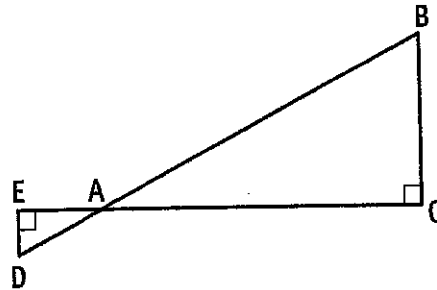
**STEP 3** Solve the proportion for  $x$ .

$$15x = 600$$

$$x = \frac{600}{15} = 40$$

**ANSWER:**  $UV = 40$  ft

**EXAMPLE 6**  $\triangle ABC \sim \triangle ADE$



Do you see that  $\triangle ABC$  is similar to  $\triangle ADE$ ?

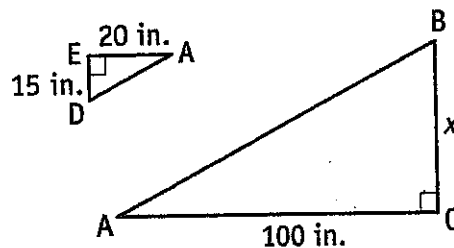
- a.  $\angle EAD = \angle BAC$  since they are vertical angles.
- b.  $\angle AED = \angle ACB$  since each is a right angle.
- c.  $\angle D = \angle B$ . Since the right angles and the vertical angles are equal, the remaining angles must be equal.

**Sample Question:**

If  $AC = 100$  in.,  $AE = 20$  in., and  $DE = 15$  in., what is the length of  $BC$ ?

**Solution:**

**STEP 1** Again, you may find it helpful to draw the similar triangles separately.



**STEP 2** Write a proportion.

$$\frac{AC}{AE} = \frac{BC}{DE} \text{ or } \frac{100}{20} = \frac{x}{15}$$

**STEP 3** Solve the proportion for  $x$ .

$$20x = 1500$$

$$x = \frac{1500}{20} = 75$$

**ANSWER:**  $BC = 75$  in.



## Right Triangles and the Pythagorean Theorem

As you have seen, a right triangle has two sides that meet in a right angle. The side opposite the right angle is called the **hypotenuse** and is the longest side of a right triangle.

The Greek mathematician Pythagoras discovered an important relationship between the hypotenuse and the other two sides. He found that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. We call this statement the **Pythagorean theorem**.

In symbols, using the labels on the triangle, you can write the Pythagorean theorem

$$c^2 = a^2 + b^2$$

If you know the lengths of the other two sides, you can find the length of the hypotenuse by following these steps.

- STEP 1** Find the square of the other two sides (often called legs) separately.
- STEP 2** Add the squares of the sides found in Step 1.
- STEP 3** Solve for the hypotenuse by finding the square root of the sum of the squares found in Step 2.

**EXAMPLE 1** What is the length of the hypotenuse of the triangle at the right?

**STEP 1** Find the square of each side separately.

a. Substitute 6 for  $a$  and find  $a^2$ .

$$a^2 = 6^2 = 36$$

b. Substitute 8 for  $b$  and find  $b^2$ .

$$b^2 = 8^2 = 64$$

**STEP 2** Add the squares found in Step 1.

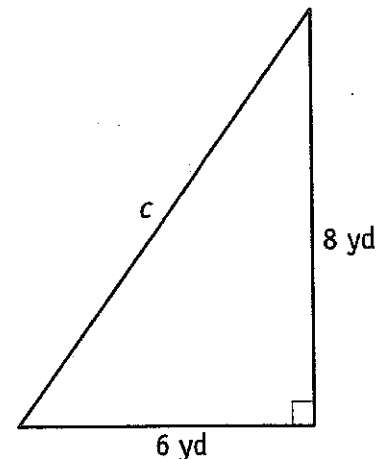
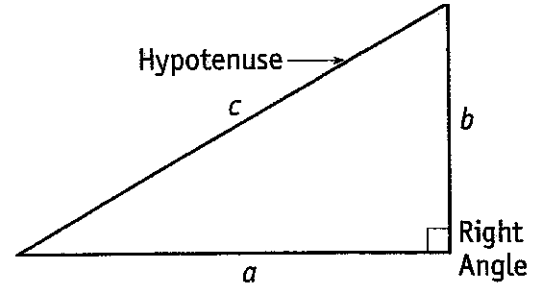
$$a^2 + b^2 = 36 + 64 = 100$$

**STEP 3** Since  $c^2 = a^2 + b^2$ , the sum of the squares found in Step 2 is equal to  $c^2$ . Then to find  $c$ , take the square root of 100.

$$c^2 = 100$$

$$c = \sqrt{100} = 10$$

**ANSWER:** The length of the hypotenuse is 10 yards.



The Pythagorean theorem can also be used to find the length of a side if the lengths of the hypotenuse and one side are known.

**EXAMPLE 2** What is the length of the unmeasured side of the triangle at the right?

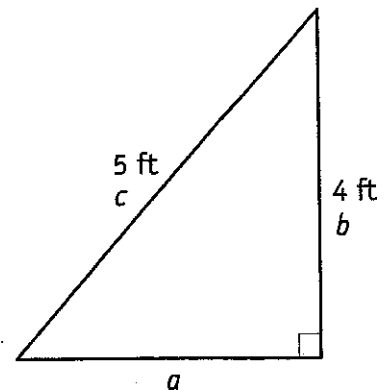
**STEP 1** Find the square of the hypotenuse. Next, find the square of the given side.

a. Substitute 5 for  $c$ , and find  $c^2$ .

$$c^2 = 5^2 = 25$$

b. Substitute 4 for  $b$ , and find  $b^2$ .

$$b^2 = 4^2 = 16$$



**Note:** You could substitute 4 for  $a$  instead of  $b$ . A measured side can be called either  $a$  or  $b$ .

**STEP 2** Write the Pythagorean theorem.

$$c^2 = a^2 + b^2$$

From Step 1, substitute 25 for  $c^2$  and 16 for  $b^2$ .

$$25 = a^2 + 16$$

Notice that 25 is equal to  $a^2$  plus 16.

To find  $a^2$ , subtract 16 from 25.

$$a^2 = 25 - 16$$

$$a^2 = 9$$

**Note:** To check that you have solved for  $a^2$  correctly, notice that  $9 + 16 = 25$ .

**STEP 3** Solve for  $a$  by taking the square root of 9.

$$a^2 = 9$$

$$\text{thus, } a = \sqrt{9} = 3$$

**ANSWER:** The length of unmeasured side  $a$  is 3 feet.

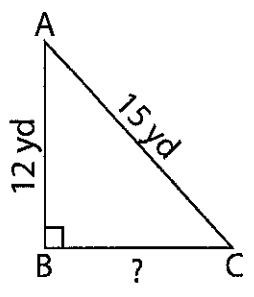
Name: \_\_\_\_\_

### Pythagorean Theorem

Sheet 1

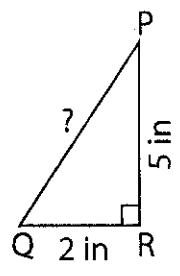
Determine the missing length in each right triangle using the Pythagorean theorem. Round the answer to the nearest tenth.

1)



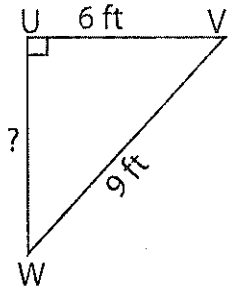
BC = \_\_\_\_\_

2)



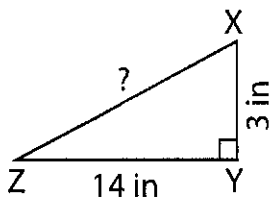
PQ = \_\_\_\_\_

3)



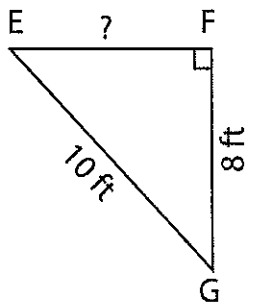
UW = \_\_\_\_\_

4)



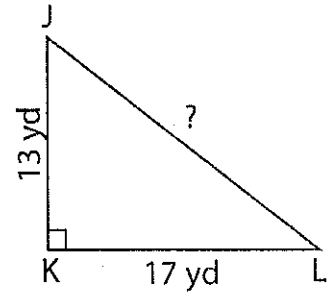
XZ = \_\_\_\_\_

5)



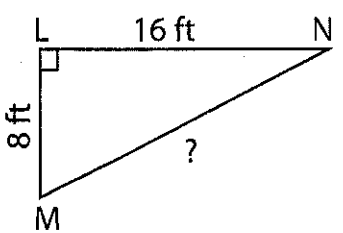
EF = \_\_\_\_\_

6)



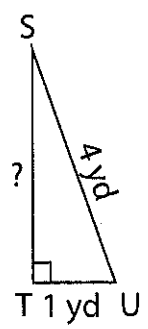
JL = \_\_\_\_\_

7)



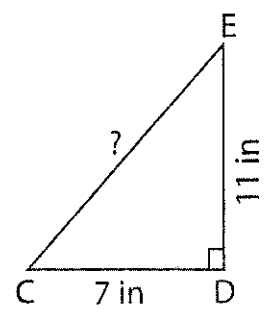
MN = \_\_\_\_\_

8)



ST = \_\_\_\_\_

9)



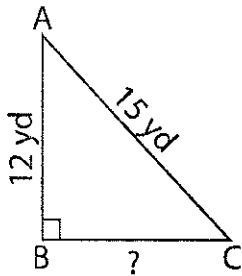
CE = \_\_\_\_\_

**Pythagorean Theorem**

Sheet 1

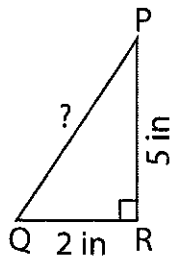
Determine the missing length in each right triangle using the Pythagorean theorem. Round the answer to the nearest tenth.

1)



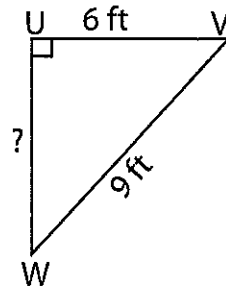
$BC = \underline{\quad 9 \text{ yd} \quad}$

2)



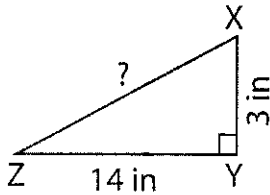
$PQ = \underline{\sqrt{29} \approx 5.4 \text{ in} \quad}$

3)



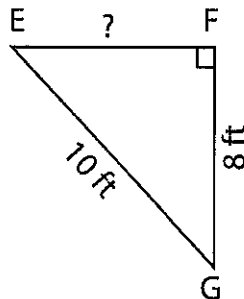
$UW = \underline{\sqrt{45} \approx 6.7 \text{ ft} \quad}$

4)



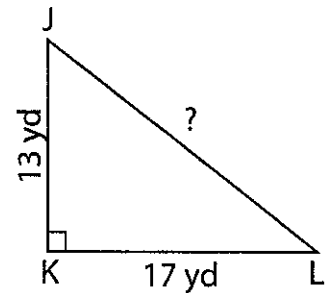
$XZ = \underline{\sqrt{205} \approx 14.3 \text{ in} \quad}$

5)



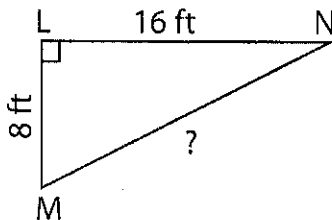
$EF = \underline{\quad 6 \text{ ft} \quad}$

6)



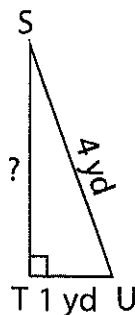
$JL = \underline{\sqrt{458} \approx 21.4 \text{ yd} \quad}$

7)



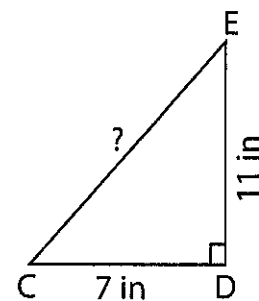
$MN = \underline{\sqrt{320} \approx 17.9 \text{ ft} \quad}$

8)



$ST = \underline{\sqrt{15} \approx 3.9 \text{ yd} \quad}$

9)



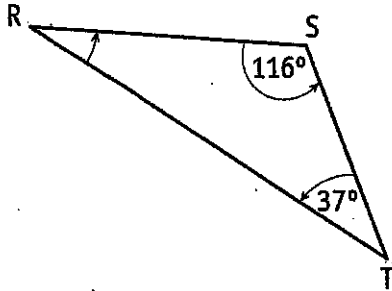
$CE = \underline{\sqrt{170} \approx 13 \text{ in} \quad}$

# Triangles Review

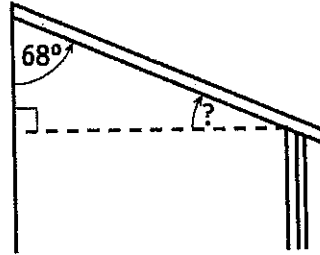
This review covers the material you have just studied. When you finish, check your answers at the back of the book.



1. What is the measure of  $\angle R$  in the triangle below?



2. What is the missing angle Bill needs to find to cut his patio roof side piece?

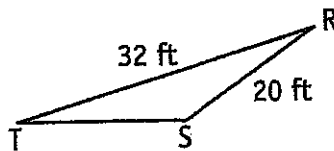
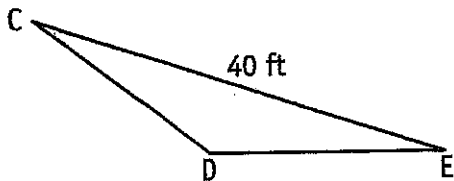


3. The sum of two angles in a triangle is  $138^\circ$ . What is the value of the third angle?

6. Solve for  $d$ .

$$\frac{10}{17} = \frac{d}{51}$$

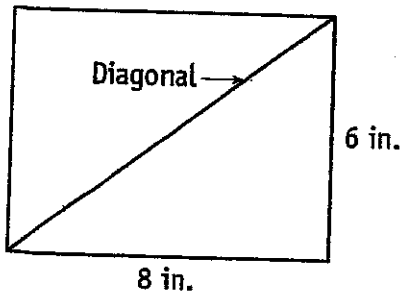
7. Triangles CDE and RST are similar triangles. What is the length of side CD in triangle CDE?



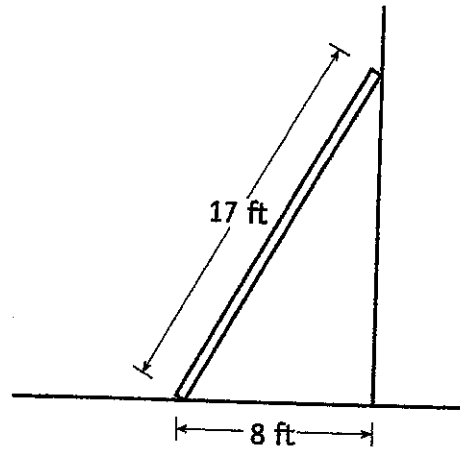
1.  $27^\circ$
2.  $22^\circ$
3.  $42^\circ$

6.  $d = 30$
7.  $CD = 25$  ft

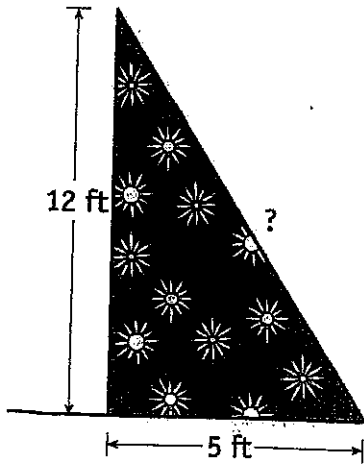
3. A rectangle is a four-sided figure with four right angles. A diagonal divides a rectangle into two right triangles. What is the length of the diagonal in the rectangle below?



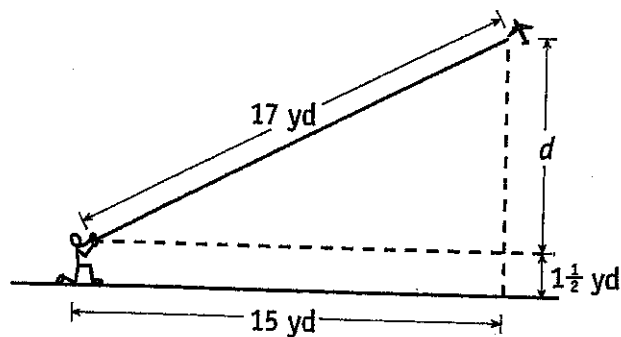
4. George leaned a 17-foot ladder against the house. The bottom of the ladder is 8 feet from the house. How high is the top of the ladder?



5. Joyce made a flower display in the front of her store. The display is in the shape of a right triangle. One side measures 5 feet. The second side measures 12 feet. How long is the side opposite the right angle?



6. Verdene is flying a model airplane at the end of the control line. The line is 17 yards long. The plane is above a point on the ground that is 15 yards away from Verdene. If the controls are held at a height of  $1\frac{1}{2}$  yards, how high is the plane off the ground?  
(Hint: height =  $d + 1\frac{1}{2}$ )



3. 10 in.  
 4. 15 ft  
 5. 13 ft  
 6.  $9\frac{1}{2}$  yd